University of Bahrain

College of Information Technology
Department of Computer Science
Discrete Structures I
Second Semester 2014-2015

Test II
*** KEY SOLUTION ***

Name	
ID	
Section	
Serial	

Time: 11:00 – 12:15 PM

This exam contains 4 pages (including this cover page) and 6 questions. Check to see if any pages are missing. Enter all requested information in the first page.

You *are not allowed* to use books, notes, or mobiles

Question	Points	Score
1	5	
2	8	
3	8	
4	6	
5	8	
6	3	
Total:	38	

Answer all questions

(1) (a) [**2 points**] Give an example for each of the following statements.

(1) $\exists a, b, c \in Z^+ : (a+b) \mid c \longrightarrow (a \mid c) \wedge (b \mid c)$.

> **Solution:** $a = 3, b = 6$, **and** $c = 18$**.** Or choose values for a, b, and c such that $(a+b) \nmid c$ and the statement becomes $F \rightarrow (a \mid c \vee b \mid c) \equiv T$.

(2) $\exists a, b \in Z$: if $a > b$ then $a^2 + b < 0$.

> **Solution:** $1 > -10$ **but** $1^2 + (-10) = -9 < 0$

(b) [**3 points**] Give a counter example for each of the following statements.

(1) If n is odd and prime , then $n+2$ is prime.

> **Solution: 7 is odd and prime but 9 is not prime**

(2) Given any set A, B. If $A - B = \varnothing$, then $B - A = \varnothing$.

> **Solution: Let** $A = \{1\}$ **and** $B = \{1, 2\}$**.** $A - B = \varnothing$ **but** $B - A = \{2\}$

(3) For any sets A and B, if $A \subseteq B$, then $|A \cup B| = |B| + |A|$.

> **Solution: Let** $A = \{1\}$ **and** $B = \{1, 2\}$**. So** $A \subseteq B$ **but** $A \cup B = \{1, 2\}$ **and** $|A \cup B| = 2$ **and** $|A| + |B| = 1 + 2 = 3 \neq 2$

(2) [**8 points**] For any integer n, show that if n is not divisible by 3 , then $n^2 - 9$ is not divisible by 3. Use *direct proof* with cases.

Solution: Case 1 : $n = 3k + 1$ $k \in Z$.
so $n^2-9 = (3k+1)^2-9 = 9k^2+6k+1-9 =$. so $3(3k^2+2k-3)+1$. let $w = (3k^2+2k-3)$.
So $n^2 - 9 = 3w + 1$ is not divisible by 3.

Case 2 : $n = 3k + 2$ $k \in Z$.
so $n^2-9 = (3k+2)^2-9 = 9k^2+12k+4-9 = 9k^2+12k-5-1+1$. so $3(3k^2+4k-2)+1$.
let $w = (3k^2 + 4k - 2)$. So $n^2 - 9 = 3w + 1$. not divisible by 3.

(3) [**8 points**] For any real number x, if $-4 \leq x \leq 4$, then $x^2 + x - 20 \leq 0$.
Prove by *contrapositive*.

Solution: contrapositive: if $x^2 + x - 20 > 0$, then $x < -4$ or $x > 4$.
$(x + 5)(x - 4) > 0$. there is two cases.
case 1 : $(x + 5) > 0$ and $(x - 4) > 0$. So $x > -5$ and $x > 4$. So $x > 4$.
case 2 : $(x + 5) < 0$ and $(x - 4) < 0$. So $x < -5$ and $x < 4$.
So $x < -5 < -4$. So $x < -4$

(4) [6 points] Prove By contradiction that for any integer a : if $a^2 - 1$ is odd , then a is even.

> **Solution:** Suppose there is an integer a such that $a^2 - 1$ is odd and a is odd. So Let $a^2 - 1 = 2h + 1$ and $a = 2k + 1$ for $k, h \in Z$. $a^2 - 1 = 2h + 1 = (2k + 1)^2 - 1 = 4k^2 + 4k + 1 - 1 = 2(2k^2 + 2k)$. Let $w = 2k^2 + 2k$. So $a^2 - 1 = 2w = 2h + 1$.
> which is even=odd contradiction. Therefore, a is even.

(5) [8 points] Let $U = \{x \in Z^+ | -5 \leq x \leq 5\}$. $A = \{x \in U | x^2 - 3x - 18 = 0\}$.
$B = \{3, 5\}$ and $C = \{1, 2, 3\}$. Answer the following. Show your work.

(a) List the elements of U and A.

> **Solution:** $U = \{1, 2, 3, 4, 5\}$
> $x^2 - 3x - 18 = (x + 3)(x - 6) = 0$. **So** $x = -3, x = 6 \notin U$. **So** $A = \varnothing$

(b) $(A \cap C) \cup B$

> **Solution:** $(\varnothing \cap \{1, 2, 3\}) \cup \{3, 5\} = \{3, 5\}$

(c) $\overline{(B \cup C)} - \overline{A}$

> **Solution:** $\{3, 5\} \cup \{1, , 2, 3\} = \{1, 2, 3, 5\} = \{1, 2, 3, 5\}$
> $\overline{\{3, 5\} \cup \{1, 2, 3\}} = \{4\}$. $\overline{A} = U$
> **So** $\overline{(B \cup C)} - \overline{A} = \{4\}) - \{1, 2, 3, 4, 5\} = \varnothing$

(d) $P(B - A)$

> **Solution:** $B - A = B = \{3, 5\} \implies \therefore P(B) = \{\varnothing, \{3\}, \{5\}, \{3, 5\}\}$

(6) [3 points] Suppose $A \subseteq B$ and $B \subseteq C$. Draw Venn Diagram for $A \cup (C - B)$

> **Solution:**